

The NaN Calculus

Eston Schweickart and Stephen McDowell

What are NaNs?

- ??
- Well, we know what they're not
 - Numbers
 - Useful
 - ...Or are they??

Who cares about NaNs?

- Concatenation of NaNs [Batman et. al 2008]
- Indian Restaurants
- Scientific computing nerds, probably
- Basically no one else

Why does no one care about NaNs?

- Mathematically useless
 - $\text{NaN} + x = x + \text{NaN} = \text{NaN}$ for all x of all types
 - NB: not in JavaScript (???)
- No formal mathematical model for NaNs
 - This is where we come in!

Anatomy of a NaN

NaNatomy

- IEEE 754 Floating Point:
 - 0 00000000 000000000000000000000000000000
 - Sign, exponent, mantissa
 - Inf if exponent is 11111111, everything else 0
 - NaN if exponent is 11111111, mantissa (payload) is nonzero
 - Many different NaNs!

Some DefiNaNtions

- $\text{NaN}_a = \text{NaN}$ with payload = a
- $\text{NaN} = \{\text{NaN}_a \mid a \text{ in } \{0,1\}^{23}\}$
- $\text{NaN}_+ = \text{NaN} \cup \{\text{Inf}\}$

Something Useful?

- $NaN+$ and $+$ form a non-abelian monoid!
- $NaN_a + x = NaN_a \quad \forall x \in NaN+$
- Inf is the identity element
-nope, still useless

Okay, what now?

- IEEE 754 limits the number of payloads
- What if there were more NaNs?
 - We can only represent a subset of them in FP
 - Distinction between float NaNs and double NaNs confirms this
- Let's create a continuous space over NaNs!

Some More Definitions

- $\mathbf{NaN} = \{\text{NaN}_\alpha \mid \alpha \in \mathbf{R}\}$
- Now we have a bijection between \mathbf{NaN} and \mathbf{R} !
- \mathbf{R} is a manifold, so \mathbf{NaN} is too.
 - A NaNifold.

The NaN Calculus

- Define all operations over NaN_a to be the same as the operation over a
- Corollary: We can perform calculus on NaNs! The NaN Calculus is the same as calculus over \mathbf{R} , but in \mathbf{NaN} instead of \mathbf{R} !
- ...Yeah, it's completely useless
 - Avenue for future work: find a use for the NaN Calculus

Conclusion

- NaNs are the worst
- Nobody cares about them
- Let them die please
- I can't get the Batman theme song out of my head

QuestionNaNs?

Backup Talk

HyperScript: Multidimensional Notation

or:

What If FEZ and LaTeX Had a
Love Child?

2D Notation is Limited

- What's the most complicated single notation symbol you can imagine?

$$\alpha \overset{\cdot}{\underset{\gamma}{\underline{\underline{M}}}}_{\lambda}^2$$

- That sucks. Not nearly convoluted enough.

Solution: More Dimensions!

- We have 3, why not use all of them
 - Seems obvious in retrospect
- Increases number of decorations by an order of magnitude!
 - 4 spaces becomes 16!
 - Space above/below character doubles!

Live Demo!!

(But not actually, because you're reading through the slides of a talk that already happened. I'm not going to embed a fracking video in the PDF because, despite it being AT LEAST 2018, videos and PDFs go together about as well as ketchup and mayonnaise. Which is to say, some people do it, I guess, but....ewwwwww. Anyway, there's a screenshot of the demo on the next slide. You're welcome.)



Future Work

- 4D and beyond
- Other implementations
 - Pop-up book
 - 3D printing
 - VR and AR

Questions????????????

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